Modal Analysis of the Magic Tee

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Abstract— **The full wave modal S-matrix of the magic-T-junction is derived by applying the mode-matching technique. Four standing wave solutions in suitably chosen subregions for the four side arms are superimposed in the common cavity region in the middle section. This yields rapid convergence and accurate results since the electric wall boundary conditions in the waveguide ports are met rigorously. The theory is verified by measurements at a magic-T-junction with standard X-band (WR 90) rectangular waveguide ports.**

I. Introduction

Due to its intrinsic power split, isolation and phase reversal characteristics, the magic-T-junction [1], [2] is a very attractive microwave circuit element for many applications such as in balanced mixers [3], E-H tuners [4], frequency discriminator circuits or directional couplers [5]. Since the circuit is not inherently matched, however, its bandwidth is limited to about 5–10% additional concentrated tuning elements are required to ensure good port VSWR's [2]. It is desirable, therefore, to utilize adequate design methods for optimizing the wavelength related dimensions of related tuning elements. As the mode-matching technique has turned out to be a very efficient design tool for such circuits, cf. e.g., [6], a useful first step towards the development of an adequate CAD program is to derive the key-building block modal S-matrix of the magic-T-junction.

Although rectangular waveguide E- and H-plane T-junctions have already been the subject for using rigorous techniques [6], the magic-T-junction has not yet been investigated by the mode-matching method so far. This letter presents the full wave modal S-matrix solution of the magic-T-junction (Fig. 1) by applying the mode-matching technique. The combination with already known key-building block modal S-matrices, i.e., for the double-step discontinuity [7], rectangular iris or post elements [6], allows then the rigorous description of adequately composed waveguide circuits. The results for the modal S-matrix of the magic-T-junction derived in this letter are verified by measurements at an X-band prototype.

II. Theory

The full-wave modal S-matrix of the magic-T-junction is derived by applying the mode-matching procedure for suitably chosen subregions (Fig. 1), like in the case of the T-junction [6].

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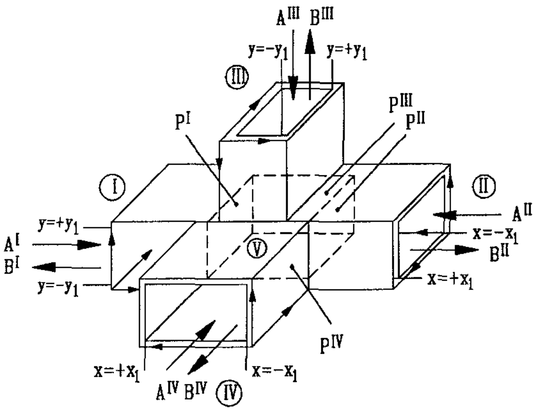

Fig. 1. Magic-T-junction.

For the waveguide subregion ν under consideration, the fields [6]

$$\vec{E}^{\nu} = \frac{1}{j\omega\epsilon}\nabla \times \nabla \times \vec{A}_e^{\nu} + \nabla \times \vec{A}_h^{\nu}$$
$$\vec{H}^{\nu} = -\frac{1}{j\omega\mu}\nabla \times \nabla \times \vec{A}_h^{\nu} + \nabla \times \vec{A}_e^{\nu} \quad (1)$$

are derived from the z components of the electric and magnetic vector potentials $\vec{A}_e, \vec{A}_h$, respectively,

$$\vec{A}_{hz} = \sum_{i=0}^{N_h} Q_{hi}T_{hi}\left[A_{hi}e^{-\gamma_{hi}z} + B_{hi}e^{+\gamma_{hi}z}\right]$$
$$\vec{A}_{ez} = \sum_{i=1}^{N_e} Q_{ei}T_{ei}\left[A_{ei}e^{-\gamma_{ei}z} - B_{ei}e^{+\gamma_{ei}z}\right], \quad (2a)$$

where A_i, B_i are the still unknown eigenmode amplitude coefficients of the forward (–) and backward (+) waves in z direction, $\gamma_{h,e}$ are the propagation factors of the N_h and N_e considered TE_{pq} and TM_{pq} modes, respectively, where i stands for p, q, Q is a normalization factor, so that the power carried by each mode is 1 W for propagating modes, j W for evanescent TE modes, $-j$ W for evanescent TM modes [6], and T are the cross-section eigenfunctions

$$T_{hi} = N_p N_q \sqrt{Z_h} \cos\left(\frac{p\pi}{a}x\right)\cos\left(\frac{q\pi}{b}y\right) \quad (3a)$$
$$T_{ei} = \sqrt{Y_e} \sin\left(\frac{p\pi}{a}x\right)\sin\left(\frac{q\pi}{b}y\right), \quad (3b)$$

with

$$N_{p,q} = \frac{1}{\sqrt{1+\delta_{0p,q}}},$$

where δ is the Kronecker delta, and Z_h, Y_e are the wave impedances or admittances, respectively.

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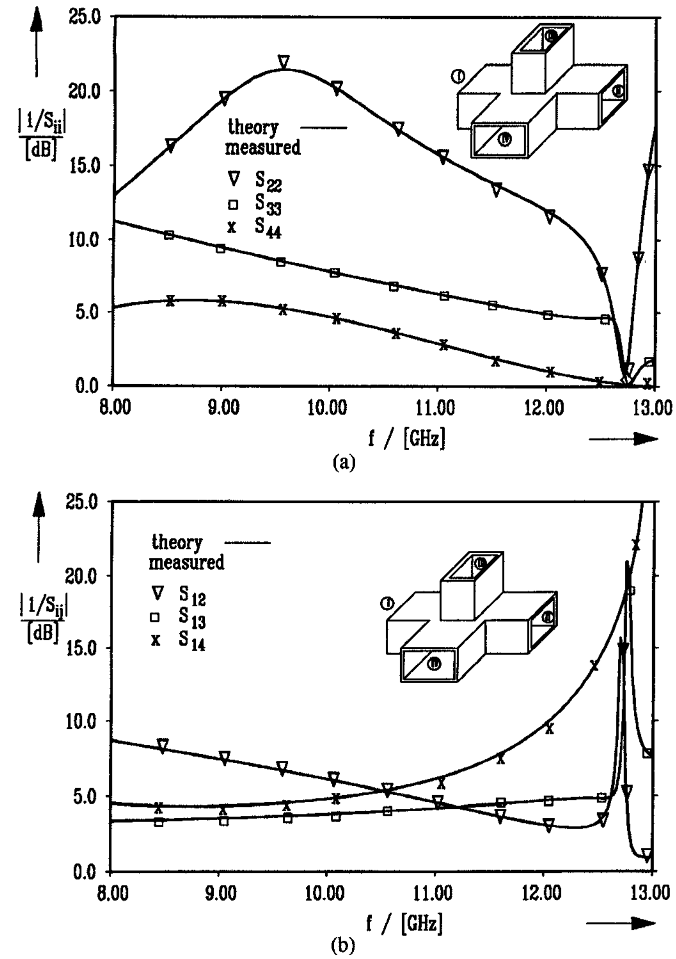

Fig. 2. Calculated (solid lines) and measured ($\nabla, \square, x$) amplitudes of the scattering parameters of a standard X-band magic-T-junction, WR90 waveguide housing (22.86 mm × 10.16 mm). (a) Input reflection coefficients. (b) Transmission coefficients.

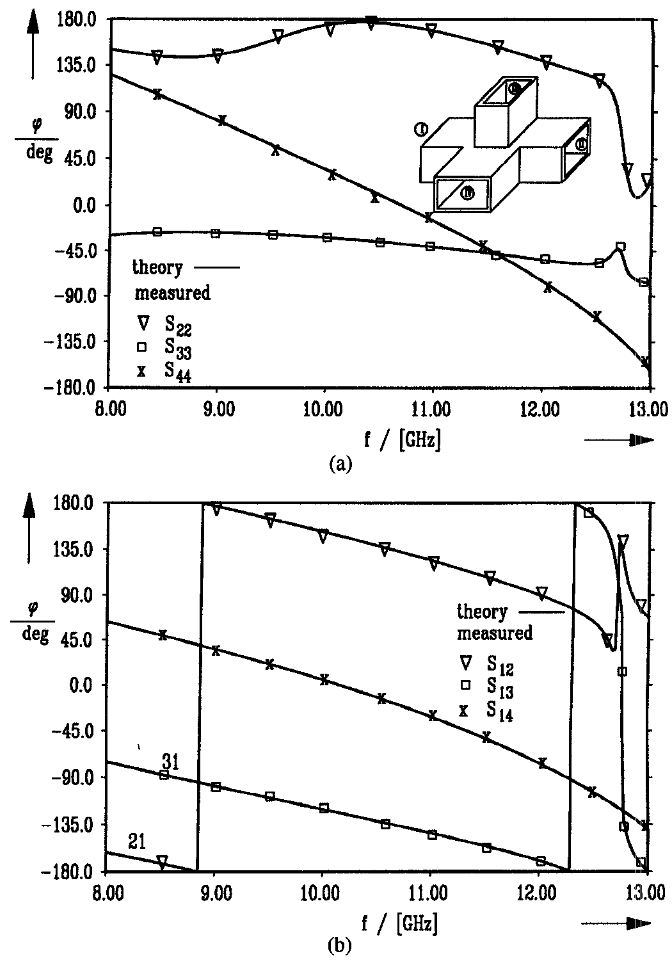

Fig. 3. Calculated (solid lines) and measured ($\nabla, \square, x$) phases of the scattering parameters of the standard X-band magic-T-junction. (a) Input reflection coefficients. (b) Transmission coefficients.

First, the field in the cavity subregion V (Fig. 1) with quadratic port waveguides of the dimensions $2x_1 \times 2y_1$ is superimposed by four suitably chosen standing wave formulations [6] $A_{h,e}^{V(1)}, A_{h,e}^{V(2)}, A_{h,e}^{V(3)}, A_{h,e}^{V(4)}$, where formulation (1) is obtained from (1)–(3) if the boundary planes P_{II}, P_{III}, P_{IV} (Fig. 1) are short-circuited and P_I is open; formulations (2), (3), and (4) are found analogously.

By matching the tangential electric and magnetic field components given by (1)–(3) between the empty waveguides I, II, III, IV, and the cavity region V at the common interfaces across the boundary planes P_I, P_{II}, P_{III}, P_{IV}, respectively and utilizing the orthogonal property of the modes, the still unknown amplitude coefficients can be related to each other in the form of the desired modal scattering matrix of the magic–T–junction with quadratic output ports.

As a second step, the known modal scattering matrix of the waveguide step key-building block discontinuity [7] is connected via the generalized scattering matrix technique directly in all port planes P. This yields immediately the complete modal scattering matrix of the standard magic–T–junction with rectangular waveguide port dimensions.

For the modal analysis of the magic-T-junction, sufficient asymptotic behavior has been obtained already by consideration of TE_{mn}—and TM_{mn}—modes up to $m = 8$, $n = 8$ in all waveguide sections. This is due to the fact that the electric wall boundary conditions in the four waveguide ports are met rigorously by this standing wave superposition technique.

III. Results

For the verification of the theory, the scattering parameters of a standard magic-T-junction with X-band waveguide (8.212.4 GHz) ports, i.e., WR90 waveguide housing dimensions (22.86 mm × 10.16 mm), are calculated and compared with measurements (Figs. 2, 3) at a very accurately fabricated prototype. The fabrication of the magic-T-junction and the measurements were carried out by ANT Nachrichtentechnik, Backnang, Germany.

Fig 2. shows the calculated amplitude (solid lines) of the input reflection coefficients (Fig. 2(a)) and transmission coefficients (Fig. 2(b)), together with the measured results (symbols $\nabla, \square, x$). The measured amplitude of S_{34} was less than –50 dB within the chosen frequency range and is not presented in the figures. Excellent agreement may be stated between the theory and the measurements, even beyond the fundamental mode cutoff frequencies. This is also true for the still more critical phases (Fig. 3) of the corresponding scattering coefficients.

IV. Conclusion

The mode matching method presented in this letter achieves the accurate analysis of the magic-T-junction. The superposition of four standing wave solutions in the common cavity region yields rapid convergence since the electric wall boundary conditions in the waveguide ports are met rigorously. Arbitrary port dimensions may be included by combining the related double-plane step discontinuity key-building blocks directly in the port planes via the generalized modal scattering matrix technique. the theory is verified by excellent agreement with measurements at a magic-T-junction with standard X-band (WR 90) rectangular waveguide ports.

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[1] N. Marcuvitz, *Waveguide Handbook.* New York: McGraw-Hill, 1951.
[2] C.G. Montgomery, R. H. Dicke, and E.M. Purcell, *Principles of Microwave Circuits.* New York: McGraw-Hill, 1948.
[3] S. March, *Microwave Mixers.* Norwood, MA: Artech House, 1985.
[4] F.E. Gardiol, *Introduction to Microwaves.* Norwood, MA: Artech House, 1984.
[5] K. Chang, Ed., *Handbook of Microwave and Optical Components*, Vol. I. New York: John Wiley, 1989.
[6] T. Sieverding, and F. Arndt, "Field theoretic CAD of open or aperture matched T-junction coupled rectangular waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 353–362, Feb. 1992.
[7] H. Patzelt, and F. Arndt, "Double-plane steps in rectangular waveguides and their application for transformers, irises, and filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 771–776, May 1982.